Tackling the Sonar Equation

$2\alpha r$ TS

V_o

G_{tvg}

G_{rec}

SL

G_1

$2D(\phi,\theta)$

$40\log(r)$

Sonar Equation: Single Target

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

V_o = voltage out (also EL echo level)

SL = transducer source level (at a specific transmit level)

G_1 = through system gain, at 1m

TS = target strength (acoustic size)

$D_i(\phi, \theta)$ = directivity index (i.e. 0 dB for on-axis targets)

40 log(r) = two-way transmission (spreading) loss at range r

α = absorption coefficient

G_{tvg} = time-varied-gain (20 or 40 log(r))

G_{rec} = receiver gain

Source Level Cal Measurement

$$SL = 20\log(i_{p\text{-}p}/8) + S_i$$

where:

$i_{p\text{-}p}$ = peak to peak current to transducer

S_i = transducer transmitting response

(pressure on axis at 1 m produced by 1 unit electrical power (units amps))

Source Level in sonar equation is a pressure from a source (p_o)

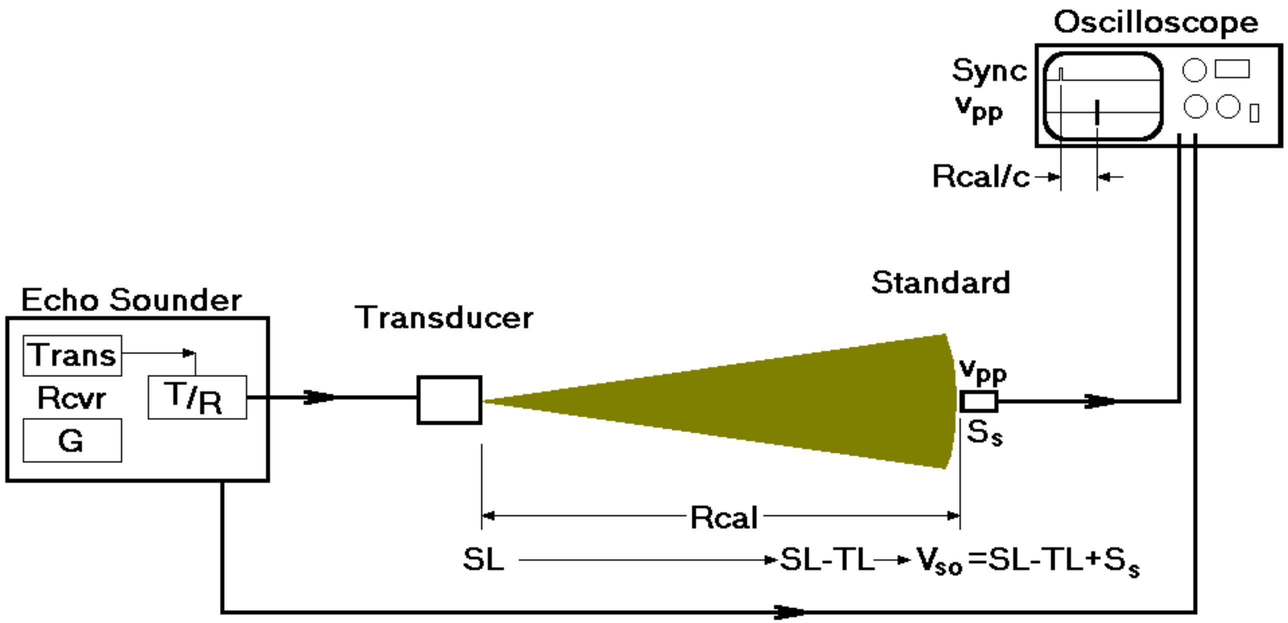
$$SL = 20\log(p_o)$$

Example:

$i_{p\text{-}p} = 40$ A $\quad S_i = 209$ dB || 1 μPa

$SL = 20\log(40/8) + 209 = 223$ dB || 1 μPa

Source Level Measurement



The oscilloscope vpp (volts) is converted to V_{so} (dB_V):

$$V_{so} = 20 \cdot \log(v_{pp}/2/1.414)$$

The Sonar equation for the one-way transmission to the standard:

$$V_{so} = SL - TL + S_s$$

$$TL_{cal} = 20 \cdot \log(R_{cal}) + \alpha R_{cal}$$

SS is a calibration value provided with the standard, therefore:

$$SL = V_{so} + TLcal - S_s$$

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

- $\mathbf{V_o}$ = voltage out (also EL echo level)
- **SL** = transducer source level (at a specific transmit level)
- $\mathbf{G_1}$ = through system gain, at 1m
- **TS** = target strength (acoustic size)
- $\mathbf{D_i(\phi, \theta)}$ = directivity index (i.e. 0 dB for on-axis targets)
- **40 log(r)** = two-way transmission (spreading) loss at range r
- α = absorption coefficient
- $\mathbf{G_{tvg}}$= time-varied-gain (20 or 40 log(r))
- $\mathbf{G_{rec}}$ = receiver gain

Through System Gain: G_1

- receive sensitivity of echosounder
- dependent on range compensation (i.e. 20 or 40 log TVG)

$$G_1 = V_{det} – L – 40\log(r_{cal}) + 2\alpha r_{cal} - G_{rec}$$

where:

V_{det} = voltage detected

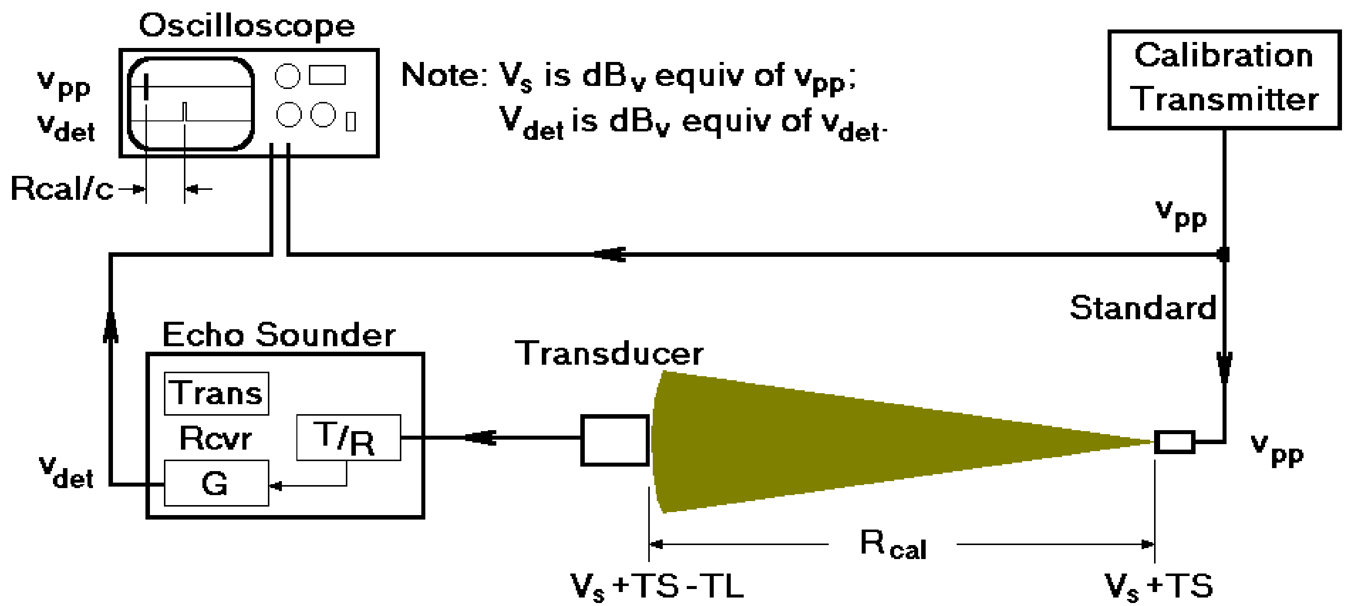
L = transducer diameter

r_{cal} = calibration range

α = absorption loss

G_{rec} = receiver gain

G_1 Measurement



$v_{det} = G + V_s + TS - TL$

$TL_{cal} = 20 \log(R_{cal}) + \alpha R$

TS is a calibration value supplied with the standard.

Remembering that $G = G_1 + G_{tvg} + RG$,

with a 40 log TVG characteristic:

$G_{tvg} = 40 \log(R_{cal}) + 2\alpha R_{cal}$

$G_1 = v_{det} - G_{tvg} - RG - V_s + TS - TL_{cal}$

Calibration Sheet: SL and G_1

Sum Channel Detected 12 kHz Output
Calibration Readings

v12kHz =	*0.408*	*volts (rms)*
Vdet =	-4.78	dB Vdet

TVG Gain G(40) = (40 log Rcal + 2a Rcal)

G(40) =	42.00	dB

Vdet = V12kHz + 3.01 dB
Sensitivity at Rcal Gx = Vdet - L

Gx =	-129.87	dB/uPa@Rcal

Sensitivity at 1 m G1 = Gx - G(40) - Rg

G1 =	-171.87	dB/uPa @ 1m

40log R G_1

20 Log R Channel Detected Output
Calibration Readings

vdet =	*0.575*	*volts (peak)*
Vdet =	-4.81	dBV (det)

TVG Gain G(20) = (20 log Rcal + 2a Rcal)

G(20) =	21.00	dB

Sensitivity at Rcal Gx = Vdet - L

Gx =	-129.90	dB/uPa@Rcal

Sensitivity at 1 m G1 = Gx - G(20) - Rg

G1 =	-150.90	dB/uPa @ 1m

20log R G_1

Transmission Loss TL = 20 log Rs + aR

TL =	15.71	dB

Source Level SL = Vso - Ss + TL

Transmit Power (dB)	Standard Transducer		Source Level (dBuPa @ 1 m)
	Vso (FFT) dBV (+20)	Vso (FFT) dBV (+40)	
20.0	-8.13		216.79
14.0	-14.06		210.86
8.0		0.01	204.93
2.0		-6.56	198.36

Source Level (SL)

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

V_o = voltage out (also EL echo level)

SL = transducer source level (at a specific transmit level)

G_1 = through system gain, at 1m

TS = target strength (acoustic size) dB re $1m^{-1}$

$D_i(\phi, \theta)$ = directivity index (i.e. 0 dB for on-axis targets)

40 log(r) = two-way transmission (spreading) loss at range r

α = absorption coefficient

G_{tvg}= time-varied-gain (20 or 40 log(r))

G_{rec} = receiver gain

Target Strength TS

- acoustic size of target (e.g. fish or zooplankton)
- ability of an object to reflect sound to the source
- linear measure: backscattering cross section σ_{bs} units m^2
- measured as a ratio of sound intensities or pressures ($I \propto p^2$)

$$\sigma_{bs} = I_r/I_i = p^2_r/p^2_i$$

$$TS = 10\log(I_r) – 10 \log(I_i) = 20\log(p_r) - 20\log(p_i)$$

$$TS = 10\log(\sigma_{bs})$$

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

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Transducer Directivity

$$D(\theta) = \frac{\sin\left(\frac{kL}{2}\sin\theta\right)}{\frac{k}{L}\sin\theta} = \text{sinc}\left(\frac{kL}{2}\sin\theta\right)$$

sinc=(sin(x)/x)

Directivity Index

$D_i = 10\log(D) = 10\log(I_o/\bar{I})$

where:

I_o = radiated intensity at acoustic axis

$\bar{I}$ = mean intensity over all directions

Calculate from transducer

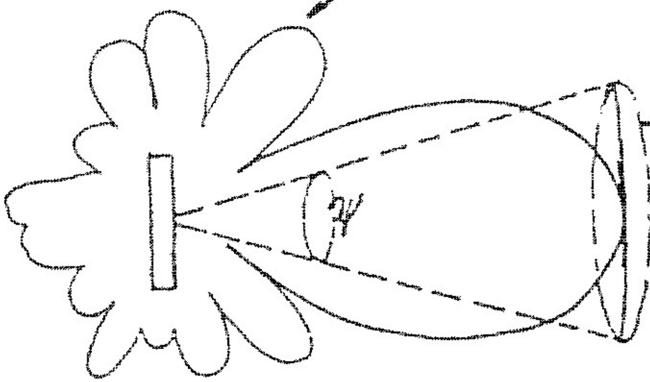
$D_i = 10\log(4\pi a/\lambda)$ where a = active transducer area

Calculate from beam angles

$D_i = 10\log(2.5/\sin(\beta_1/2)\sin(\beta_2/2))$

where β's = beam width at –3dB points

Equivalent Ideal Beam Pattern



$$\psi = \int_{4\pi} b^2 d\Omega$$

$$10\log(\Psi) = 10\log(\beta_1\beta_2/5800)$$

where β is active length of transducer

If square or circular transducer:

$$10\log(\Psi) = 10\log(\beta^2/5800)$$

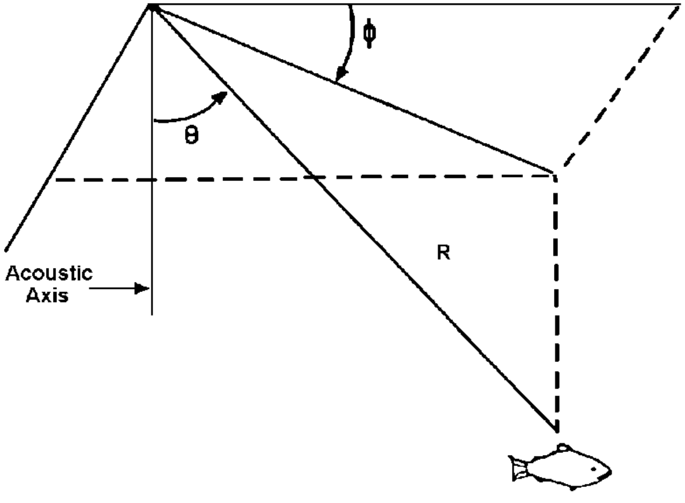
$$\psi = \left(\frac{4.853}{kD}\right)^2$$

where k is wavenumber and D diameter of transducer

Integrated Beam Pattern Factor

- one-way loss in signal intensity due to the angle of the target relative to the acoustic axis

$$I = k\,\frac{10^{-2\alpha R}}{R^4}\,b^2(\theta,\phi)\,\sigma_{bs}$$



$$TS = 10\log_{10}\sigma_{bs}$$

Effect of Beam Pattern

- transmit response (i.e. acoustic level) is highest along acoustic axis
- receive response (i.e. echo level) is highest along acoustic axis
- echo received from a target will decrease off axis due to transmit and receive losses
- echo amplitude of a target depends on acoustic size of fish and position in beam

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

V_o = voltage out (also EL echo level)

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40 log(r) = two-way transmission (spreading) loss at range r

α = absorption coefficient

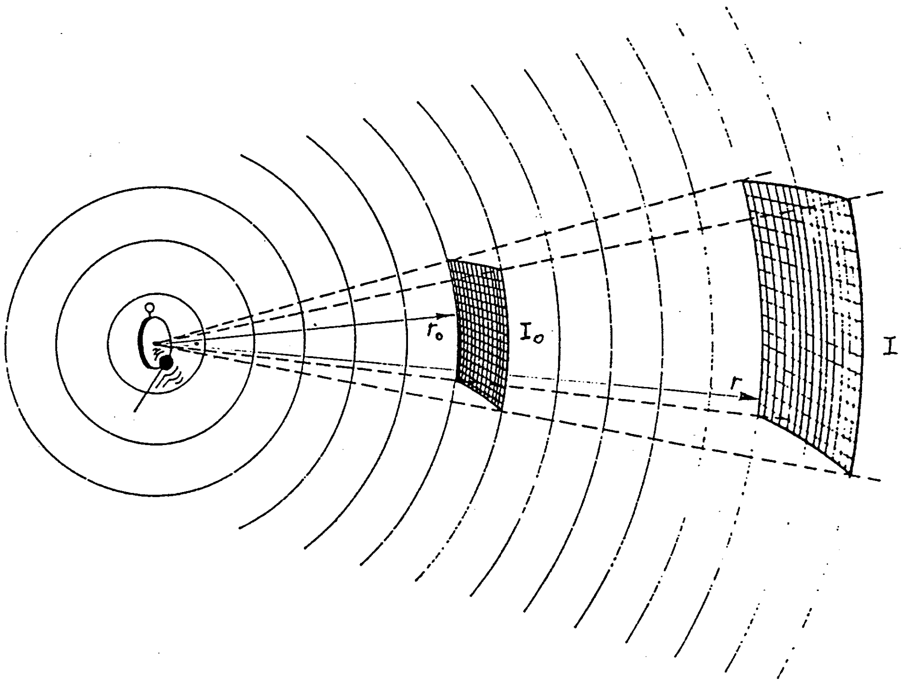
G_{tvg} = time-varied-gain (20 or 40 log(r))

G_{rec} = receiver gain

Transmission Loss

Geometric Spreading

- pressure decreases as the 1/distance from source
- spherical spreading from a point source (e.g. transducer)
- 2-way spreading increases as range^2



$$I_o/I = (r/r_o)^2$$

$$TL = 10\log(I_o/I) = 20\log(r/r_o)$$

if $r_o = 1$ m

then one way $TL = 20\log(r)$

and two way $TL = 40\log(r)$

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

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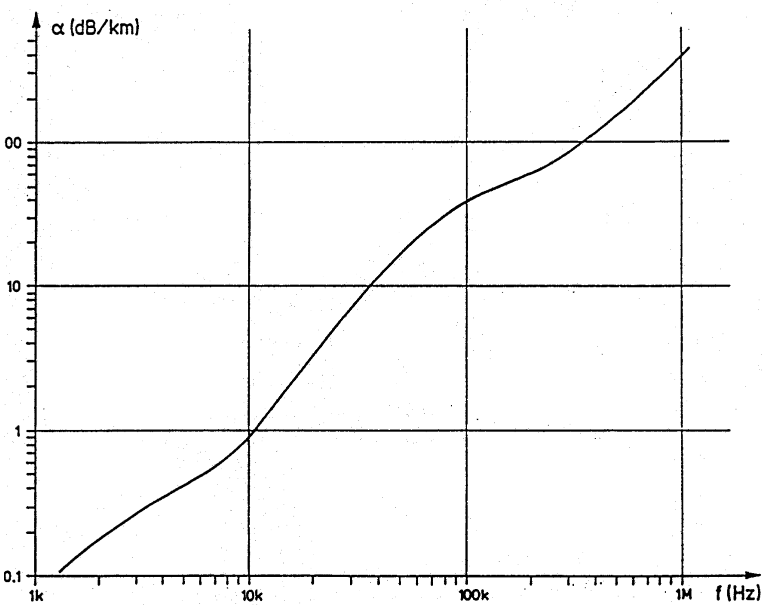
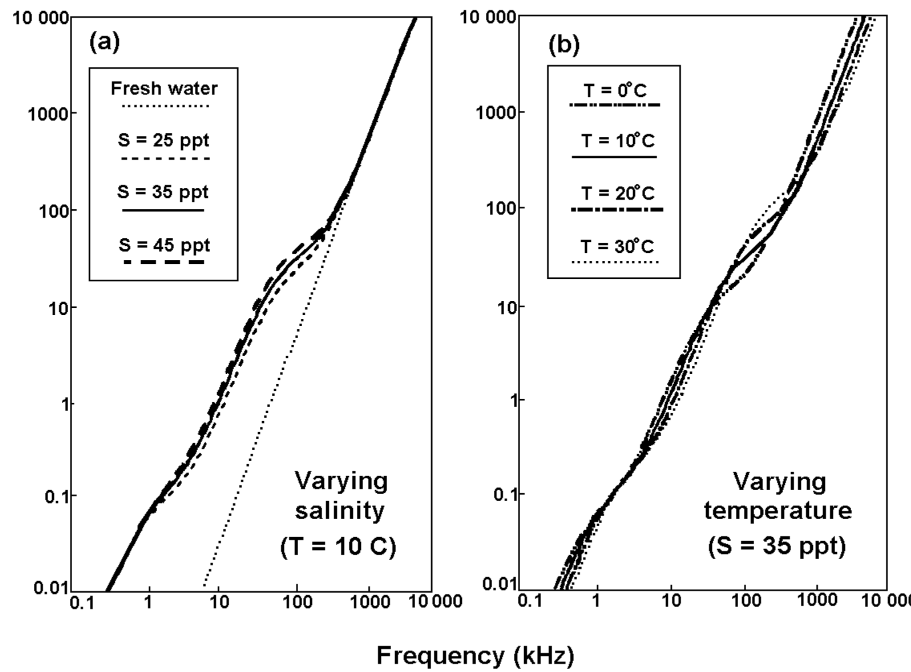
G_{rec} = receiver gain

Absorption

- attenuation of pressure due to friction (α, units nepers/m or dB/m))
- proportional to range
- dependent on frequency: increases proportional to the square of frequency
- higher in salt water than fresh water



ABSORPTION COEFFICIENT VS. FREQUENCY



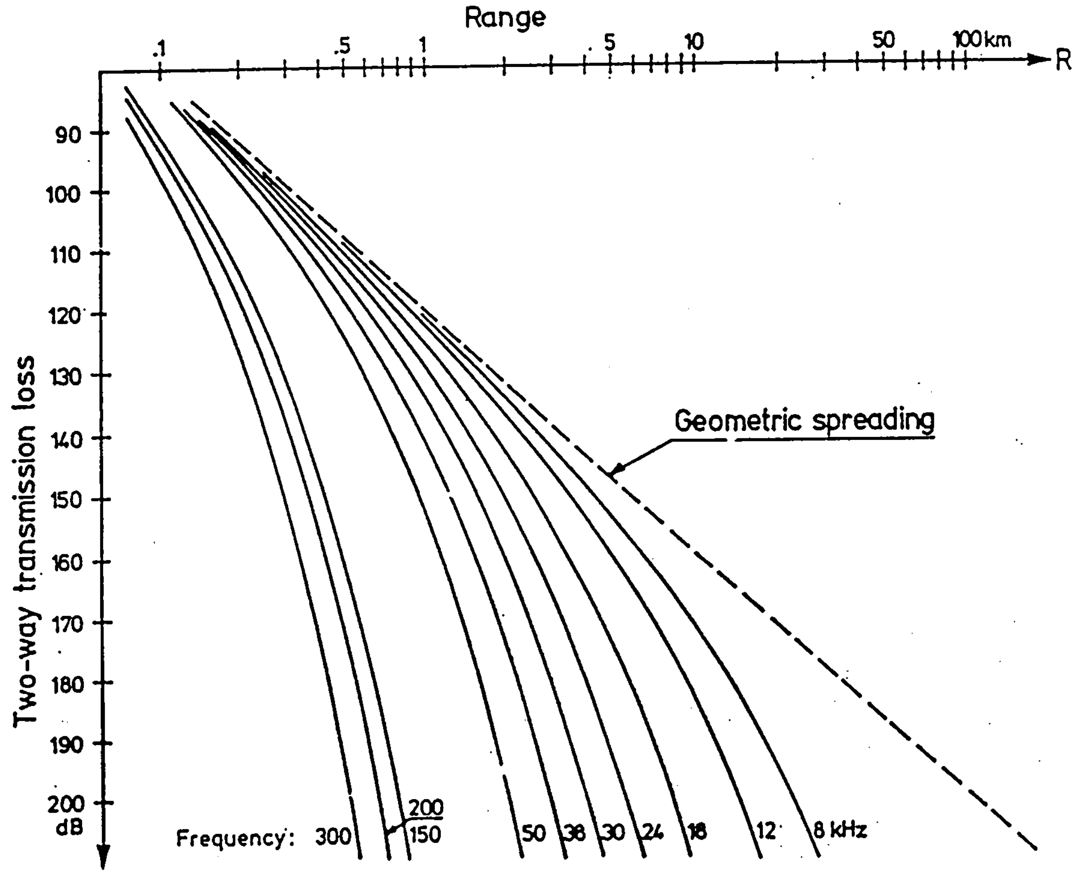
Absorption Loss

One way: αr, units dBm^{-1}

Two way: $2\alpha r$, units dBm^{-1}

Total Transmission Loss

Total transmission loss (two way): $40 \log(r) + 2\alpha r$



TWO WAY TRANSMISSION LOSS VS. FREQUENCY.

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

- V_o = voltage out (also EL echo level)
- **SL** = transducer source level (at a specific transmit level)
- G_1 = through system gain, at 1m
- **TS** = target strength (acoustic size)
- $D_i(\phi, \theta)$ = directivity index (i.e. 0 dB for on-axis targets)
- **40 log(r)** = two-way transmission (spreading) loss at range r
- α = absorption coefficient
- G_{tvg} = time-varied-gain (20 or 40 log(r))
- G_{rec} = receiver gain

Range Compensation: TVG

Time Varied Gain

- amplification applied to received echo to compensate for transmission loss due to beam spreading
- constant TVG is main reason why 'scientific' echosounders cost lots

Single target: small relative to wavelength
individual targets can be resolved (dependent on target density and pulse duration)

one way spreading loss = $1/r^2$

two way spreading loss = $1/r^4$

Log form: $10\log(r^4) = 40\log(r)$

Range Compensation: TVG

If **Multiple** targets: assume constant density
individual targets can not be resolved
spreading is range-dependent
target distribution is large relative to beam width

one way spreading loss = $1/r$

two way spreading loss = $1/r^2$

Log form: $10\log(r^2) = 20\log(r)$

Distinguishing 20 log and 40log TVG

20 log(r) (used for echo integration)

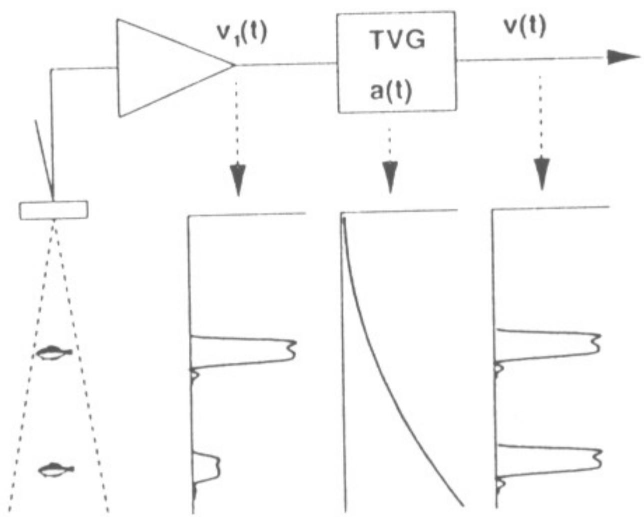
- Echo level for fish at range r α $1/r^2$
- $(\text{Echo level})^2 \;\alpha\; 1/r^4$
- # fish @ r increases with area of beam (i.e. $1/r^2$)

So, squared signal α $r^2(1/r^4) = 1/r^2$

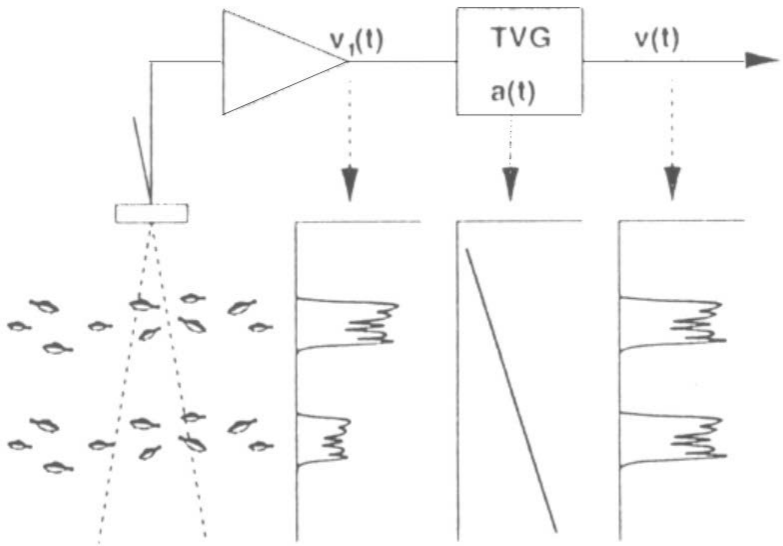
Squared signal in dB α $10\log(1/r^2) = -20\log(r)$

G_{tvg}: Time Varied Gain

Individual targets: 40log(r)



Multiple targets: 20log(r)



where $v_1(t)$ = uncompensated voltage, a(t) = receiver gain,
v(t) = compensated voltage

Sonar Equation (log form)

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

where:

V_o = voltage out (also EL echo level)

SL = transducer source level (at a specific transmit level)

G_1 = through system gain, at 1m

TS = target strength (acoustic size)

$D_i(\phi, \theta)$ = directivity index (i.e. 0 dB for on-axis targets)

40 log(r) = two-way transmission (spreading) loss at range r

α = absorption coefficient

G_{tvg}= time-varied-gain (20 or 40 log(r))

G_{rec} = receiver gain

G_{rec}: System Receiver Gain

- amplification applied to received echo to center dynamic range of echosounder
- some manufacturers user selected: range -12 dB to +12 dB
- other manufacturers user sets minimum detected and then adds range (typically 36 dB)

Sonar Equation Example

You are on the NOAA R/V Oscar Dyson in the Gulf of Alaska. You are interested in the length distribution of juvenile walleye pollock in Barnabus Trough. You have a **120 kHz echosounder** and the Traynor et al. publication that tells you: **$TS = 20\log(L_{cm}) - 66$**. You measure a target strength of **-45 dB re 1 μPa** from a fish at **100m** range. The water is **10º C** with a salinity of **35**, resulting in an absorption coefficient of **38.7 dB/km**. The system is set so that you have a source level of **216.78 dB re 1 μPa**. From the transducer calibration parameter sheet you know that the directivity index is **-5 dB re 1 μPa**, and the through system gain is **171.87 dB re 1 μPa** .

What is the voltage recorded on your echosounder and what is the length of the fish?

Juvenile Walleye Pollock Length

$$V_o = SL + G_1 + TS + 2D_i(\phi,\theta) - 40\log(r) - 2\alpha r + G_{tvg} + G_{rec}$$

Frequency = 120 kHz   Target Range = 100 m
H_2O Temp = 10°C   Salinity = 35 ppt

where:

V_o = voltage out (also EL echo level)

SL = 216.78 dB re 1 μPa transducer source level

G_1 = -171.87 dB re 1 μPa through system gain, at 1m

TS = -45 dB re 1 μPa target strength

$D_i(\phi, \theta)$ = -5 dB re 1 μPa directivity index

40 log(r) = 80 dB re 1 μPa two-way transmission loss at range r

α = 0.0387 dB/m (120 kHz, 10° C, 35 ppt) absorption coefficient

G_{tvg}= 80 dB re 1 μPa 40 log(r) time-varied-gain

G_{rec} = 0 dB re 1 μPa receiver gain

Sonar Equation Example

$$\mathbf{V_o = SL + G_1 + TS + 2D(\phi,\theta) - 40log(r) - 2\alpha r + G_{tvg} + G_{rec}}$$

$$\mathbf{V_o = 216.79 + (-171.87) + (-45) + 2(-5) - 80 - 7.74 + 80 + 0}$$

$$\mathbf{V_o = -17.82\ dB_v}$$

$20log(volts) = dB_v$ $10^{dBv/20} = volts$

$$\mathbf{10^{dBv/20} = 0.12853\ volts}$$

TS = 20log(L)-66

L = 11.22 cm